Dynamics

Newton’s Second Law of Motion

- The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.

$$\Sigma F = ma$$

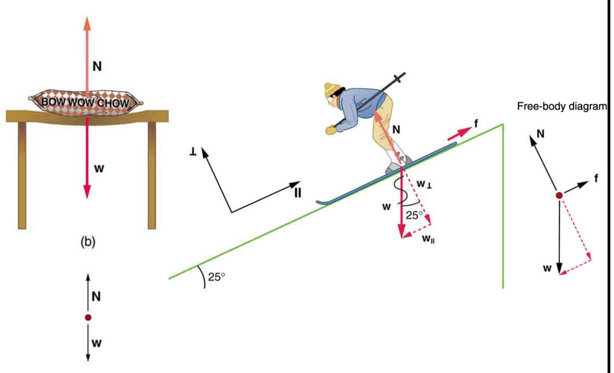
Translational Equilibrium

- An object is said to be in translational equilibrium if and only if the net force on the system is zero.
  - The vector sum of all the forces acting on the system is zero.

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma F_z = 0$$

Normal Force

- When an object is sitting on a surface the surface must support the load by exerting an upwards force equal to the weight.
- If the force supporting a load is perpendicular to the surface of contact between the load and its support, this force is defined to be a normal force.



Friction

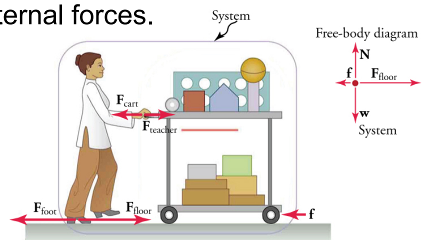
- Friction is an external force that acts opposite to the direction of relative motion or to prevent slipping.
- The magnitude of the force of friction is proportional to the normal force.

$$F_f = \mu F_N$$

μ is the coefficient of friction

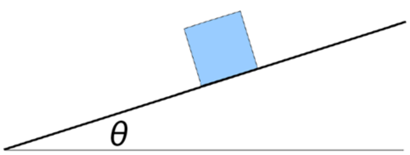
System

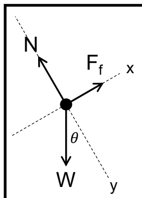
- A system is defined by the boundaries of an object or collection of objects being observed; all forces originating from outside of the system are considered external forces.



Example

- A 20. kg block is accelerating down a ramp at a rate of 0.50 m/s^2. The ramp is 30° above the horizontal. Calculate the coefficient of kinetic friction between the block and the ramp.





$$F_f = \mu F_N$$

x

$$\Sigma F = ma$$
$$-F_f + W \sin\theta = -ma$$
$$F_f = ma + mg\sin\theta$$

y

$$\Sigma F = ma$$
$$N - W\cos\theta = 0$$
$$N = mg\cos\theta$$

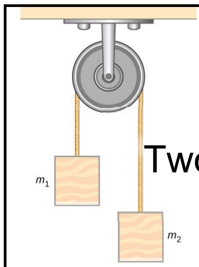
$$\mu = \frac{F_f}{F_N} = \frac{ma + mg\sin\theta}{mg\cos\theta}$$

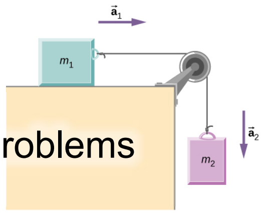
$$\mu = \frac{a + g\sin\theta}{g\cos\theta}$$

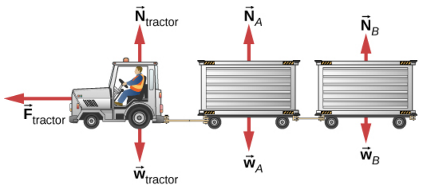
$$\mu = \frac{0.5 + 9.8\sin 30}{9.8\cos 30}$$

$$\mu = 0.64$$

Two-Body Problems

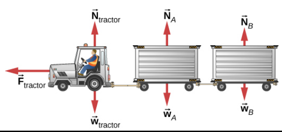




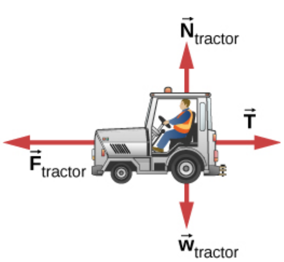


Example 1

A baggage tractor pulls luggage carts from an airplane. The tractor has mass 650.0 kg, while cart A has mass 250.0 kg and cart B has mass 150.0 kg. A driving force of 820.0 N acts for 3.00 s accelerating the system from rest. What is the horizontal force acting on the connecting cable between the tractor and cart A at this instant?



To calculate the tension between the tractor and cart A, we need to treat the tractor as a system.

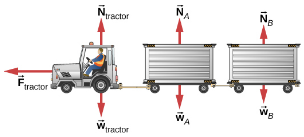


$$\Sigma F = ma$$

$$F_{tractor} - T = ma$$

$$T = F_{tractor} - ma$$

To calculate the acceleration, we need to look at the system of the tractor and the baggage cars.



$$\Sigma F = ma$$

$$F_{tractor} = (m_{tractor} + m_A + m_B)a$$

$$a = \frac{F_{tractor}}{(m_{tractor} + m_A + m_B)}$$

$$a = \frac{820}{(850 + 250 + 150)} = 0.67\ \text{m/s}^2$$

Now we can solve for the tension between the tractor and cart A.

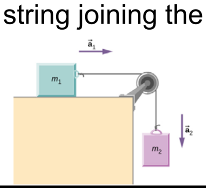
$$T = F_{tractor} - ma$$

$$T = 820 - (850)(0.67)$$

$$T = 250\ \text{N}$$

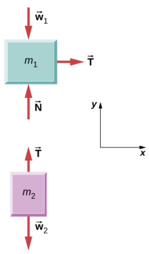
Example 2

A block on a frictionless, horizontal surface is pulled by a light string that passes over a frictionless and massless pulley. The other end of the string is connected to another block. The mass of block 1 is 1.0 kg and the mass of block 2 is 2.0 kg. Calculate the tension in the string joining the blocks.



To calculate the tension in the string we need to treat either block 1 or block 2 as a system.

Note: Block 1 accelerates in the positive x direction. Block 2 accelerates in the negative y direction.



Block 1: $\Sigma F = ma$

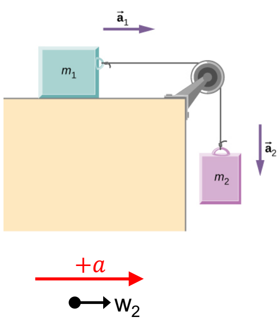
$T = m_1 a$

Block 2: $\Sigma F = ma$

$T - w_2 = m_2(-a)$

$T = -m_2 a + m_2 g$

To calculate the acceleration, we need to consider the system of both blocks.



$\Sigma F = ma$

$w_2 = (m_1 + m_2)a$

$$a = \frac{m_2 g}{(m_1 + m_2)}$$

$$a = \frac{(2)(9.8)}{(1+2)} = 6.533$$

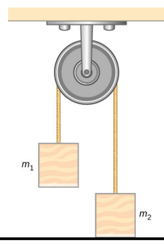
Now we can calculate the tension in the string.

Note: We can use either block 1 or block 2.

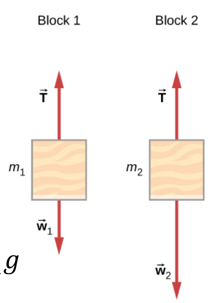
Block 1	Block 2
$T = m_1 a$	$T = -m_2 a + m_2 g$
$T = (1)(6.533)$	$T = -(2)(6.533) + (2)(9.8)$
$T = 6.5\text{ N}$	$T = 6.5\text{ N}$

Example 3

An Atwood machine consists of a rope running over a pulley, with two objects of different mass attached. If $m_1 = 2.00\ \mathrm{kg}$ and $m_2 = 4.00\ \mathrm{kg}$, what is the tension in the string?



To calculate the tension in the string we need to treat either block 1 or block 2 as a system.



Block 1:

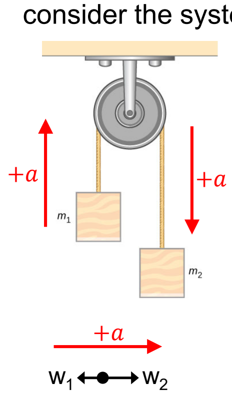
$$\Sigma F = ma$$
$$T - w_1 = m_1 a$$
$$T = m_1 a + w_1$$
$$T = m_1 a + m_1 g$$

Block 2:

$$\Sigma F = ma$$
$$T - w_2 = m_2(-a)$$
$$T = -m_2 a + w_2$$
$$T = -m_2 a + m_2 g$$

Note: the blocks accelerate in opposite directions

To calculate the acceleration, we need to consider the system of both blocks.



$$\Sigma F = ma$$
$$-w_1 + w_2 = (m_1 + m_2)a$$
$$a = \frac{-m_1 g + m_2 g}{(m_1 + m_2)}$$
$$a = \frac{(m_2 - m_1)g}{(m_1 + m_2)}$$
$$a = \frac{(4-2)(9.8)}{(2+4)} = 3.27\ \mathrm{m/s^2}$$

Now we can calculate the tension in the string.

Note: We can use either block 1 or block 2.

Block 1	Block 2
$T = m_1 a + m_1 g$	$T = -m_2 a + m_2 g$
$T = m_1(a + g)$	$T = m_2(-a + g)$
$T = 2(3.27 + 9.8)$	$T = 4(-3.27 + 9.8)$
$T = 26.1\ \mathrm{N}$	$T = 26.1\ \mathrm{N}$